EFFECT OF BRANCHING RATIO FOR NITROGEN LASER SYSTEM

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Abstract: The effect of non-vanishing branching ratio on various parameters such as unsaturated population difference N(Z), cascade transition rate (R) and the difference of population in terms of density matrix ($\rho_{aa} - \rho_{bb}$) has been estimated for nitrogen laser system. The semi-classical formula of Lamb Jr. has been used for estimation of the difference of population in terms of density matrix. The present simplified analysis indicates how the branching ratio quantitatively to be used to the basic process involved in the laser system.

Key words : nitrogen laser, branching ratio, population difference, cascade transition, density matrix.

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INTRODUCTION

According to E. Wills & Lamb Jr.[1] the lower maser level could at least in part be excited by spontaneous emission from the upper. They have discussed a term branching ratio f. Recent Borah & Barua[2] have calculated branching ratio and its effect on various parameters in case of Ar^+ laser system. Branching ratio effect on various parameters in He-Ne laser has been reported by author[3]. In present study we have calculated the branching ratio for Nitrogen laser system and shown its effect on various parameters.

Method and Result

Let the response frequency of the transition from Level 'b' to 'a'. is ω The number of atoms per unit volume per unit time excited to level 'a' and 'b' are λ_a and λ_b and the decaying rates are $\gamma_a \rho_{aa}$ and $\gamma_b \rho_{bb}$ respectively. The number making transitions[4] from the level 'a' to 'b' is $R (\rho_{aa} - \rho_{bb})$. The figure of transitions has been shown in Fig. 1
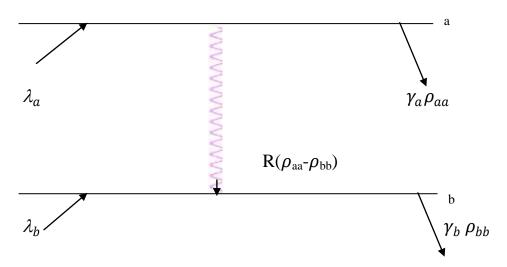

Fig. 1 :- The energy level diagram for the two level atoms comprising laser active medium.

The rate equations can be represented by

$$\dot{\rho}_{aa} = - \gamma_a \rho_{aa} + R (\rho_{bb} - \rho_{aa}) + \lambda_a$$

$$\dot{\rho}_{bb} = - \gamma_b \rho_{bb} + R (\rho_{aa} - \rho_{bb}) + \lambda_b + f \gamma_a \rho_{aa} \tag{1}$$

where the source term of f $\gamma_a \rho_{aa}$ describes the effect of radiative cascade excitation of 'b' from 'a'.

As shown in Fig. 1, In steady state the population density difference (ρ_{aa}- ρ_{bb}) becomes :

$$\rho_{aa}- \rho_{bb} = \left[\frac{\lambda_a}{\gamma_a}\left\{1 - f\left(\frac{\gamma_a}{\gamma_b}\right) - \frac{\lambda_b}{\gamma_b}\right\}\right] \times [1+ R\{\gamma_a(1 + f) + \gamma_b\}| \gamma_a\gamma_b]^{-1} \quad (2)$$

It will be seen that the effect of a nonvanishing branching ratio f is merely to change the unsaturated population difference (obtained for R = o) and also to modify the value of R for which a given value of saturation parameter will be modified in a obvious fashion. Thus if f = o, a value of $R = \frac{1}{2} \gamma_a\gamma_b/\gamma_{ab}$ would cause 50% saturation, while if f = 1 the value would be R = γ_a.

The effect of branching ratio on various parameters

We have estimated that the nitrogen laser line 337.1 nm originated[5] from C³ π_u to B³ π_g level. For this decay chain we want to calculate the value of unsaturated population difference N(Z), cascade transition rate (R) and difference of population in terms of density matrix (ρ_{aa}-ρ_{bb}) and the effect of branching ratio f on these parameters.

We know that, the branching ratio

$$f = \frac{A_{ij}}{\sum_j A_{ij}} = \frac{A_{ij}}{A_i} = \frac{decay\ from\ upper\ level\ to\ lower\ level}{Total\ decay\ from\ upper\ level} \quad (3)$$

Where, A_{ij} is the partial transition probability, A_i is the total probability for level i.

Now for decay chain, $C^3\pi_u$ - $B^3\pi_g$ i.e. for 337.1 nm line, the value of transition probabilities[5-6]

$$A_i = \frac{1}{t_i} = \frac{1}{10\times10^{-6}} \text{ Sec} = 10^5/_{\text{sec}} \tag{4}$$

$$\text{Similarly, } A_j = \frac{1}{t_j} = \frac{1}{40\times10^{-9}} \text{ Sec} = 2.5\times10^7/\text{sec} \tag{5}$$

If n_i and n_j are the upper and lower level population and A_i & A_j are total probabilities for level i & j and τ_i and τ_j are the lifetimes[7]. We known that

$$\frac{n_i}{n_j} = \frac{A_j}{A_{ij}} \tag{6}$$

Putting the values[8] of n_i & n_j in the above equation we obtained the value of A_{ij}

$$A_{ij} = 1.1627\times10^3/\text{sec} \tag{7}$$

Where, A_{ij} is the partial transition probability

hence, the branching ratio $f = \frac{A_{ij}}{A_i} = 0.01163$ (8)

The inversion density[8] $(\frac{n_i}{g_2} - \frac{n_j}{g_1})$ of levels between which oscillation in N₂ laser is observed, where, n_i and n_j are the upper and lower level populations, and g_2 and g_1 are the usual statistical weight[9] of the upper and lower levels respectively (at pressure 80 Torr and pulse repetition rate 15HZ) is given by

$$\frac{n_i}{g_2} - \frac{n_j}{g_1} = 21.499\times10^{13} \tag{9}$$

as $g_1 = g_2 = 1$

Hence, $n_i - n_j = 21.499\times10^{13}$ (10)

Now, we can write the decay constants

$$\gamma_a = A_i = \frac{1}{10 \times 10^{-6 \text{ sec}}} = 10^5 \text{ S}^{-1} \tag{11}$$

$$\gamma_b = A_j = \frac{1}{40 \times 10^{-9 \text{sec}}} = 2.5 \times 10^7 S^{-1} \tag{12}$$

And the upper state population

$$n_i = 2.1 \times 10^{14} \tag{13}$$

$$\therefore \lambda_a = \frac{n_i}{\tau_i} = 2.15 \times 10^{19} \text{ S}^{-1} \tag{14}$$

Similarly, $\lambda_b = \frac{n_i}{\tau_j} = 0.0537 \times 10^{23}$ S^{-1} \tag{15}

Hence, the unsaturated population difference

$$\text{N(Z)} = \left[\frac{\lambda_a}{\gamma_a} \left\{ 1 - f\left(\frac{\gamma_a}{\gamma_b} \right) - \frac{\lambda_b}{\gamma_b} \right] = 0.000925 \times 10^{14} \tag{16}$$

But the value of unsaturated population difference without f

$$\text{N(Z)} = \lambda_a \gamma_a{}^{-1} - \lambda_b \gamma_b{}^{-1} = 21.499 \times 10^{13} \tag{17}$$

Here the above it is apparent that the decrease in the value of unsaturated population difference is due to the presence of branching ratio. This also indicates gain in population of the lower level via excitation of this level through spontaneous decay from upper levels.

The cascade transition rate[10],

$$\text{R} = \lambda_a \sum \tau\, C^3\, \pi_\mu A C^3 \pi_u\, B^3 \pi_g$$

$$= (2.15 \times 10^{19})\, (10 \times 10^{-6})\, (1.1627 \times 10^{13})$$

$$= 24.99805 \times 10^{26} \tag{18}$$

Where $\tau c^3\, \pi_u$ is life time. $AC^3\pi_u$, $B^3\, \pi_g$, Einstein A Coefficient connecting the states expressed in the subscript.

Therefore the denominator of the eq. (2), becomes

$$1 + \text{R} \frac{\{\gamma_a(1+f)+\gamma_b\}}{\gamma_a\gamma_b} = 25.097 \times 10^{21} \tag{19}$$

Now, the difference of population in terms of density matrix,

$$\rho_a\rho_a - \rho_b\rho_b = \left\{ \frac{\gamma_a/\gamma_a(1-f\ \gamma_a/\gamma_b)- \lambda_a\gamma_b}{1+\frac{R\{\gamma_a(1-f)+\gamma_b\}}{\gamma_a\gamma_b}} \right\} = 0.0856642 \times 10^{-7} \tag{20}$$

This value of population difference in terms of density matrix is quite realistic and shows substantial inversion at the desired level.

Conclusion

It is obvious from the present calculation that the effect of non-vanishing branching ratio f is to decrease the unsaturated population difference and hence to increase the excitation of the lower level.

The present study provides the use of the concept of branching ratio to analyse its effect on various parameters involved in the scheme in a quantitative manner. It is also apparent that the present calculation is simple but significant in this regard.

The value of population difference[3] in terms of density matrix in case of He-Ne laser system is 0.713 whereas in case of nitrogen laser system is 0.086×10^{-7} according to the present calculation. This value is very less in comparison to the value 0.713 for He-Ne laser system. It shows one of the season why He-Ne laser is continuous laser whereas Nitrogen laser in a pulse laser.

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REFERENCES

1. Lamb Jr., W.E., *Phys. Rev*. **A** 1449 (1964).

2. P. Borah and G.D. Baruah. *Ph.D. Thesis, Dibrugarh University, Dibrugarh, Assam,* 127 (1994)

3. Nagmani Prasad Sinha and P. Poddar, *Journal of Pure Applied and Industrial Physics*, **9** (2), 8 (2019)

4. Smith and Sorakin, Laser *Mc-Graw Hill Physical Quantum electronics series*, 21 (1966).

5. S.K. Tiwari & D.K. Rai, *Laser and its Applications*, Wawaswat Press, Varanasi, 61 (1997).

6. P. Borah and G.D. Baruah. *Ph.D*. *Thesis, Dibrugarh, University, Dibrugarh, Assam* 138 (1994).

7. S.K. Tiwari & D.K. Rai, *Laser and its Applications*, Wawaswat Press, Varanasi, 64 (1997).

8. P. Persephonics, *Rev. Phys Appl*. **21**, 609-614 (1986).

9. Pearson and Gordon, *Molecular spectra*, 310 (1977).

10. P. Borah and G.D. Baruah, *Ph.D. Thesis Dibrugarh University, Dibrugarh, Assam*, 137. (1994).